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MULTISHAKER MODAL TESTING

by

Dr. Roy R. Craig, Jr.

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MULTISHAKER MODAL TESTING

Final Report

Contract No. NAS8-33980
August 1980 - October 1983

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Marshall Space Flight Center, AL 35812

by

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ABSTRACT

This report summarizes the research conducted on this contract and previously reported in a number of technical reports, papers presented at technical meetings, and published papers. The principal goal of the contract was to explore procedures for improving the modal modeling of structures using test data and to determine appropriate analytical models based on substructure experimental data.

In the area of modal modeling using test data, two related research topics were considered: modal modeling using several independently-acquired columns of frequency response data, and modal modeling using simultaneous multi-point excitation. In the area of component mode synthesis modeling, the emphasis has been on determining the best way to employ complex modes and residuals. This report presents abstracts of the major publications which have been previously issued on these topics.

MULTISHAKER TEST METHODS

The period of this contract has been a period of great activity on the topic of multishaker modal testing. On the one hand, Vold and his colleagues at Structural Dynamics Research Corporation developed a multi-input modal estimation algorithm for minicomputers (1-3) which simultaneously employs several columns of frequency response function (FRF) data. On the other hand, Allemang (4,5) developed a simultaneous multi-input method for computing (single-input) columns of the FRF matrix. A combination of the multi-input modal estimation algorithm and the Allemang algorithm for using simultaneous uncorrelated multiple inputs to generate the FRF's required by the modal estimation algorithm has led to a great improvement in experimental modal modeling technology.

The goals of the present contract in the area of multishaker test methods are along lines similar to the work of Vold and Allemang mentioned above. However, on the topic of modal modeling using several independently-acquired columns of FRF data, research has followed a different path than that taken by Vold. In Report CAR 81-1 (6) and Report CAR 82-1 (7) the "Asher method" and a generalization of the Asher method called the "minimum coincident response method" were employed to determine real modes and undamped natural frequencies from independently-acquired columns of FRF data. An application of multishaker random testing is also included in Report CAR 81-1. The modal tuning work was also summarized in a paper presented at the March 1982 SDRC Troubleshooting Software User's Conference (8). Abstracts of these reports and papers are presented below.

A parameter estimation algorithm which employs multi-input excitation directly has been recently developed and is described fully in Report CAR 83-1 (9). A paper summarizing this work has been submitted to the AIAA Dynamics Specialists Conference (10). With this algorithm it is not necessary to gener-

ate columns of the FRF matrix prior to entering a modal parameter estimation algorithm as is done in the Allemang/Vold procedure.

COMPONENT MODE SYNTHESIS

One of the ultimate goals of refined modal test methods is to produce math models of structural components which are sufficiently accurate to employ in synthesizing system models from component models. Component mode synthesis research has been conducted concurrently with the modal test research summarized above in order to clarify which component modal parameters are ultimately needed for synthesizing accurate system models.

Chung developed both Hamiltonian and state vector methods of component mode synthesis. These are described in his Ph.D. dissertation (11). The Hamiltonian formulation is summarized in References 12 and 13, while the state vector formulation is summarized in Reference 14. A more simplified state vector component mode synthesis formulation has been developed by Howsman. The work completed to date has been submitted as a technical paper to the AIAA/ASME/ASCE/AHS 25th Structures, Structural Dynamics and Materials Conference (15). Work on this topic is being continued under NASA Contract NAS8-35338.

ABSTRACTS OF REPORTS AND TECHNICAL PAPERS

Modal Analysis Using a Fourier Analyzer, Curve-Fitting, and Modal Tuning (Ref. 6)

This report proposes a modal testing procedure including the following tasks: (1) data acquisition and FFT processing, (2) curve-fitting of single FRF's, (3) modal tuning, (4) mathematical modeling, and/or (5) computer-controlled testing. Steps (1) through (3) are described in the report.

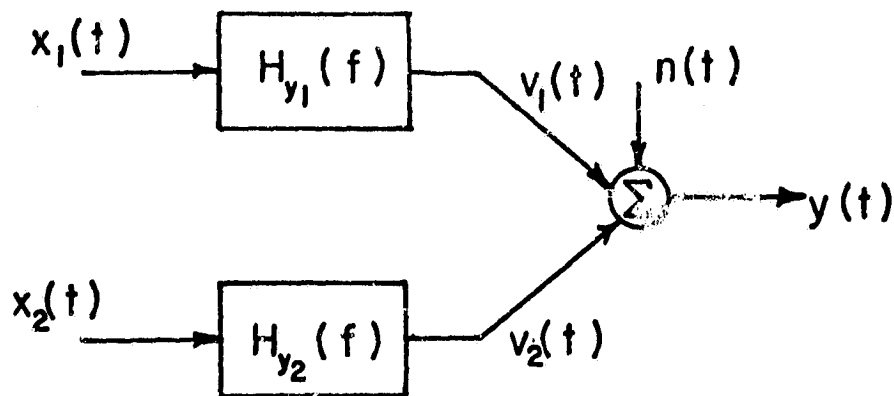


Figure 1. Two-Input, Single-Output System

The multishaker procedure for acquiring FRF's which was proposed by Allemang (4,5) is outlined in the report. For example, for the two-input, single-output system shown in Fig. 1, the Fourier-transformed response $Y(f)$ is given by

$$Y(f) = H_{y1}(f) X_1(f) + H_{y2}(f) X_2(f) + N(f) \quad (1)$$

The least squares estimates for H_{y1} and H_{y2} are given by

$$\begin{aligned} H_{y1} &= \frac{G_{y1}G_{22} - G_{y2}G_{21}}{G_{11}G_{22} - |G_{12}|^2} = \left(\frac{G_{y1}}{G_{11}} \right) \left[\frac{1 - \frac{G_{y2}G_{21}}{G_{y1}G_{22}}}{1 - \gamma_{12}^2} \right] \\ H_{y2} &= \frac{G_{11}G_{y2} - G_{12}G_{y1}}{G_{11}G_{22} - |G_{12}|^2} = \left(\frac{G_{y2}}{G_{22}} \right) \left[\frac{1 - \frac{G_{y1}G_{12}}{G_{y2}G_{11}}}{1 - \lambda_{12}^2} \right] \end{aligned} \quad (2)$$

where

$$G_{y1} = YX_1^*, \quad G_{11} = X_1X_1^*, \quad \text{etc.} \quad (3)$$

and γ_{12} is the ordinary coherence function between inputs $X_1(f)$ and $X_2(f)$.



Figure 2. Dual-Beam with Modified Ends and Modified Shaker Attachment

Dual-shaker uncorrelated random excitation was applied to the dual-beam test structure of Fig. 2. The time histories from two force cells and four accelerometers were tape recorded and subsequently played back through a HP5420A Fourier analyzer to compute the necessary auto- and cross-spectra. Although 25 records were averaged in computing these auto- and cross-spectra, the computed FRF's were not of acceptable quality. This appears to be attributable to the use of tape-recorded data, which did not permit proper time correlation of the records input to the two-channel analyzer. Because of the failure of this attempt to employ multiple shakers, efforts were renewed to secure funding for a 16-channel modal analysis system, and subsequent testing was confined to 2-channel processing of single-shaker FRF's.

The curve-fitting of individual FRF's was performed in order to obtain the best analytical fit of the FRF's over limited frequency ranges. The MDOF curve-fit algorithm (GE command) in the MODAL PLUS program of Structural Dynamics Research Corporation was employed.

In order to provide a rational procedure for incorporating multiple FRF's in the determination of system modal parameters, two modal tuning algorithms were employed, the "Asher method" of Ref. 16 and the "minimum coincident response method" of Ref. 17. In the Asher method, approximations to the true undamped natural frequencies of a system are obtained by computing the roots of the determinant of the real part of the FRF matrix.

$$\det [\hat{C}(\hat{f}_c)] = 0 \quad (4)$$

This method requires a square FRF matrix, i.e. as many exciters as responses. The minimum coincident response method permits the number of response DOF's to exceed the number of excitation DOF's by selecting frequencies and force distributions which minimize an error function defined as the sum of the squares of the coincident (real) responses, i.e.

$$\epsilon = \tilde{Y}_R^T \tilde{Y}_R = X^T [\tilde{C}]^T [\tilde{C}] X \quad (5)$$

The basic algorithms are presented in Ref. (6), while applications are presented in Ref. (7).

Modal Vector Estimation for Closely-Spaced-Frequency Modes (Ref. 7)

The purpose of this report is to discuss the difficulty of obtaining accurate mode shapes of systems with closely-spaced natural frequencies, and to present examples of the use of modal tuning to obtain modal parameters of such systems by simultaneously using several FRF's. Table 1 illustrates the fact that while frequencies of a system are only moderately sensitive to system changes, the mode shapes are extremely sensitive to small stiffness or mass changes.

m'	f_1	ϕ_1	f_2	ϕ_2
1.000	1.0000	1.000 1.000	1.0002	1.0000 -1.0000
1.001	0.9994	1.0000 1.6176	1.0016	-1.6192 1.0000
1.010	0.9910	1.0000 10.0098	1.0011	-10.1099 1.0000
1.100	0.9100	1.0000 91.0100	1.0010	-100.1109 1.0000

Table 1. Effect of Subsystem Properties on System Modes of a Weakly Coupled 2DOF System

Thus, while modal tuning methods which are based on multiple FRF's can be expected to produce improved frequency estimates, it may be expected that mode shapes will be more difficult to determine accurately.

Complete derivations of the Asher method and the minimum coincident response method are given in the report. Both methods were applied to determine the frequencies and mode shapes of the weakly-coupled beam structure of Fig. 2. The effect of analysis bandwidth on modes and frequencies was studied for the two modes at 118.80 Hz and 119.03 Hz.

The conclusions drawn from this report are (1) that the standard Asher method and minimum coincident response method are rational procedures for employing multiple FRF's for identifying modal parameters, and (2) that lightly-coupled systems will always lead to great difficulty in obtaining "accurate" mode shapes because of the extreme sensitivity of the mode shapes to system parameters.

Modal Vector Estimation for Closely-Spaced-Frequency Modes (Ref. 8)

This paper, presented at the GE/CAE International Troubleshooting Software User's Conference, summarizes the work presented in Refs. 6 and 7 and abstracted above.

A Generalized Multiple-Input, Multiple-Output Modal Parameter Estimation Algorithm (Refs. 9,10)

A new multi-shaker modal analysis algorithm for estimation of modal coefficients has been developed in a thesis (Ref. 9) and summarized in a technical paper (Ref. 10). The algorithm permits multiple inputs (exciter locations) as well as multiple outputs (accelerometer locations) to be employed. The excitation may be simultaneous random or simultaneous swept sine. Free decay responses may also be used. The algorithm is an extension of an algorithm

published by Coppolino in Ref. 18. Measurement stations are designated as "independent" or "dependent," with the number of independent stations being equal to the number of modes to be identified in a given frequency range. Modal vectors, which are assumed to be complex, are defined at both independent and dependent degrees of freedom.

The algorithm is based on the theory that a linear structure may be accurately described over a limited frequency range by a finite number of degrees of freedom. The frequency domain equation relating output acceleration to input forces is

$$\{\ddot{x}(\omega)\} = [H(\omega)] \{F(\omega)\} \quad (6)$$

It may be rewritten in a form which identifies independent and dependent outputs (accelerations) and input forces as follows:

$$\{\ddot{x}_i\} = [\phi_i] [A(\omega)] [\phi_i]^T [D_{id}] \{f(\omega)\} \quad (7a)$$

$$\{\ddot{x}_d\} = [\phi_d] [\phi_i]^{-1} \{\ddot{x}_i\} \quad (7b)$$

Equation (7a) is transformed into an equation for the n independent coordinates

$$[M_i] \{\ddot{x}_i(\omega)\} + [C_i] \{\dot{x}_i(\omega)\} + [K_i] \{x_i(\omega)\} = [D_{id}] \{f(\omega)\} \quad (8)$$

Accelerations are determined at k frequencies ($k > n$) and assembled into a matrix

$$[\ddot{x}_i(\omega)] = [\ddot{x}_i(\omega_1) \mid \ddot{x}_i(\omega_2) \mid \cdots \mid \ddot{x}_i(\omega_k)] \quad (9)$$

Equation (8) may be rewritten for each of the k frequencies and assembled in the form

$$-[\ddot{x}_i] = [\tilde{C}_i] [\dot{x}_i] - [\tilde{D}_{ia}] \begin{bmatrix} [\dot{x}_i] \\ [x_i] \\ [f] \end{bmatrix} \quad (10)$$

where $\{\dot{x}_i(\omega)\}$ and $\{x_i(\omega)\}$ are obtained from the measured $\{\ddot{x}_i(\omega)\}$ by the equations

$$\{\dot{x}_i(\omega)\} = -\frac{j}{\omega} \{\ddot{x}_i(\omega)\} \quad (11a)$$

$$\{x_i(\omega)\} = -\frac{1}{\omega^2} \{\ddot{x}_i(\omega)\} \quad (11b)$$

Equation (10) is solved in a least-squares manner for $[\tilde{C}_i]$ and $[\tilde{K}_i]$, which are used to form an eigenequation whose solution gives the natural frequency and damping estimates and the $[\phi_i]$ portion of the modal vectors. Equation (7b) is used to recover the $[\phi_d]$ portion of the modal vectors.

Simulation studies were performed to test the applicability and accuracy of the algorithm. In one study, random forces were applied and accelerations measured at all 8 DOF's of an 8 DOF system having closely-spaced-frequency modes in the range 5 Hz to 15 Hz. The analysis range was taken as 0 Hz to 1024 Hz with 256 data points, for a delta frequency of 4 Hz. All modes were identified to extremely high accuracy in spite of the fact that there were seven natural frequencies between two of the data points. No noise was introduced into the data used in this study.

More realistic simulation studies were carried out on a 9 DOF system using 1, 4, 5, and 8 inputs; 5 and 8 independent DOF's; pure random and random-phased swept sine excitation; and noise ratios of 0%, 10%, and 20%. Table 2 shows the frequencies, damping factors, and mode shapes of the 9 DOF simulation model. Tables 3 and 4 show the identified modes corresponding to the 5 Hz, 5.5 Hz and 5.55 Hz modes. Eight independent DOF's were employed. Note that the single exciter in Table 3 was unable to identify the 5 Hz mode because

of its location near a "node line" of the 5 Hz mode, while the 4-exciter results in Table 4 show acceptable frequencies and mode shapes for all three modes.

Mode No.	Frequency (Hz)	Damping
1	5.000	0.010
2	5.250	0.010
3	5.500	0.010
4	5.550	0.010
5	5.900	0.010
6	6.200	0.010
7	6.600	0.010
8	15.000	0.010
9	25.000	0.010

MODE	DOF	1	2	3	4	5	6	7	8
1		.010	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2		1.000	.010	1.000	1.000	-1.000	-1.000	-1.000	-1.000
3		1.000	1.000	-.010	-1.000	-1.000	-1.000	1.000	1.000
4		1.000	1.000	-1.000	-.010	1.000	1.000	-1.000	-1.000
5		1.000	-1.000	-1.000	1.000	.010	-1.000	-1.000	1.000
6		1.000	-1.000	-1.000	1.000	-1.000	.010	1.000	-1.000
7		1.000	-1.000	1.000	-1.000	-1.000	1.000	-.010	1.000
8		1.000	-1.000	1.000	-1.000	1.000	-1.000	1.000	-.010
9		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2. System Frequencies, Damping Factors, and Mode Shapes
(defined at 8 DOF only)

No mode corresponding to 5 Hz was identified

Undamped Natural Frequency = .549815E+01
Damping = .922684E-02

Coordinate	Coefficient	
1	.100000E+01	0
2	.961693E+00	.154095E-01
3	-.111589E+00	-.243182E+00
4	-.953094E+00	-.189219E-01
5	-.866331E+00	.201760E+00
6	-.874222E+00	.225422E+00
7	.852086E+00	-.247594E+00
8	.852671E+00	-.202553E+00

Undamped Natural Frequency = .556574E+01
Damping = .125684E-01

Coordinate	Coefficient	
1	.969798E+00	.718633E-02
2	.100000E+01	0
3	-.601954E+00	.299185E+00
4	-.306331E-01	.115159E+00
5	.663191E+00	-.152995E+00
6	.655221E+00	-.180120E+00
7	-.574999E+00	.258952E+00
8	-.641525E+00	.178636E+00

Force at DOF No. 1 Noise Ratio = 10%

Table 3. Single-Exciter Results

Undamped Natural Frequency = .499588E+01
Damping = .101836E-01

Coordinate	Coefficient	
1	.180826E-01	.64856E-02
2	.989374E+00	.422064E-03
3	.984854E+00	.601076E-02
4	.100000E+01	0
5	.975333E+00	.364940E-02
6	.987170E+00	-.791027E-02
7	.977551E+00	.291086E-02
8	.975623E+00	-.161876E-03

Undamped Natural Frequency = .549835E+01
Damping = .979900E-02

Coordinate	Coefficient	
1	-.995591E+00	.353937E-01
2	-.963411E+00	.140834E-01
3	-.193424E-01	-.112736E-01
4	.987221E+00	.125764E-01
5	.952534E+00	-.241061E-01
6	.100000E+01	0
7	-.995252E+00	-.490844E-01
8	-.960618E+00	-.491432E-01

Undamped Natural Frequency = .554825E+01
Damping = .943557E-02

Coordinate	Coefficient	
1	.993486E+00	-.268232E-01
2	.100000E+01	0
3	-.973952E+00	.261893E-02
4	-.554699E-01	.508959E-02
5	.852456E+00	-.601461E-02
6	.903672E+00	-.198642E-02
7	-.925322E+00	-.291869E-01
8	-.905322E+00	-.464260E-01

Forces at DOF's 1,2,3 and 4

Noise Ratio 10%

Table 4. Four-Exciter Results

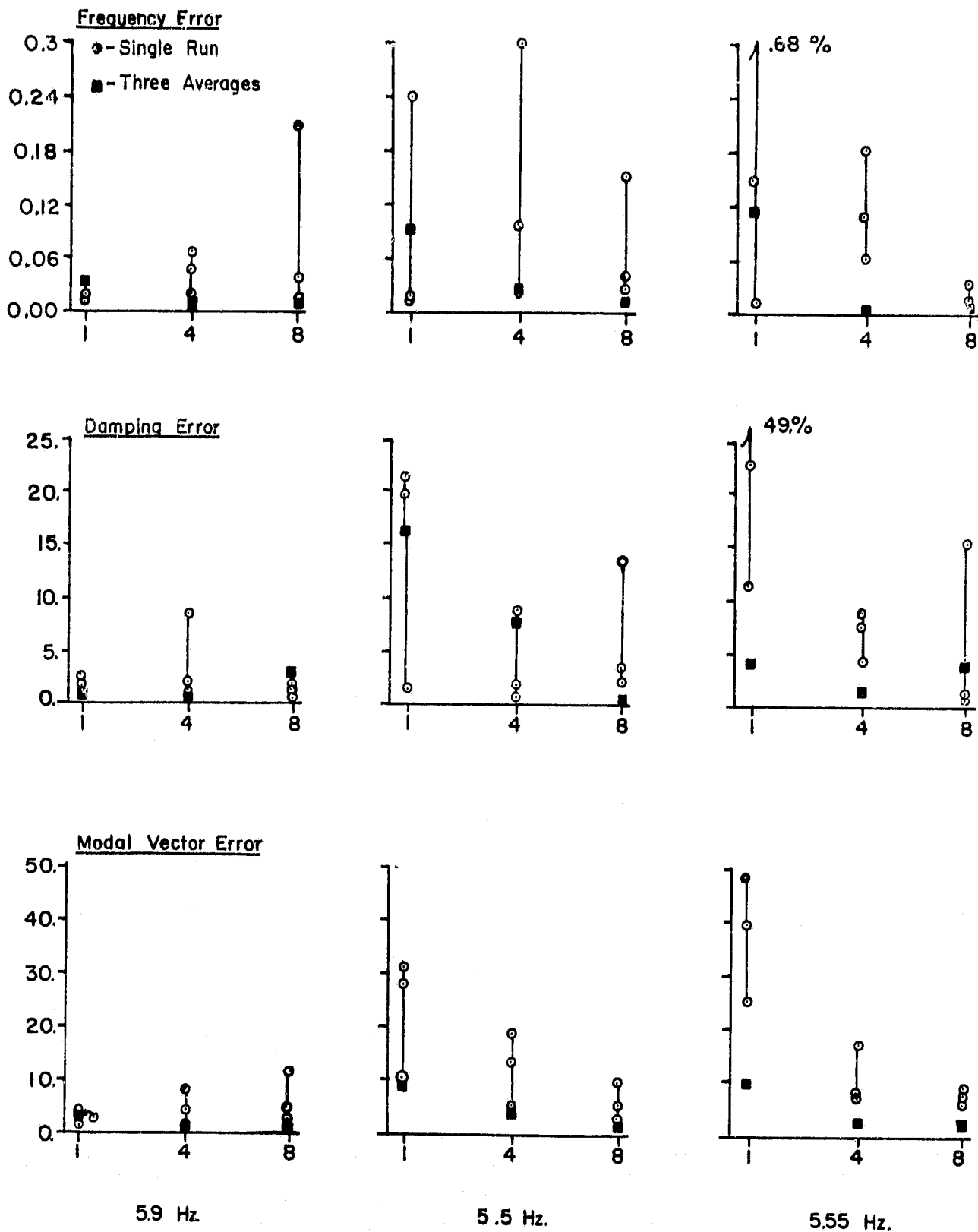


Figure 3. Modal Parameter Errors Versus Number of Shakers
for Single Runs and for 3-Run Average

The results in Tables 3 and 4 were obtained from single "runs." Averaging may be employed to improve the parameter (frequency, damping, mode shape) estimates. Figure 3 illustrates the results of single runs with random excitation and the results of three "runs" averaged.

This algorithm meets the primary objective of this NASA contract, which was to produce an improved method for multi-shaker modal testing. Although testing of this algorithm is still in progress, it is believed that it will prove to be a valuable addition to the multi-input modal identification software which is currently in great demand in the modal testing industry.

Application and Experimental Determination of Substructure Coupling for Damped Structural Systems (Ref. 11)

The principal topic covered in this thesis is the development of a generalized substructure coupling procedure for a complex structure with general viscous damping. A Hamiltonian first-order differential equation formulation is employed in order to permit complex substructure modes to be employed easily, since complex modes are required for systems with non-proportional damping. The substructure coupling procedure makes use of an incomplete set of complex normal modes in conjunction with complex residual attachment modes to account for the contribution of neglected higher-frequency modes. Also discussed in the dissertation are experimental procedures for identifying the substructure data by modal testing.

The first-order substructure, or component, equations of motion are

$$a\dot{y} + by = \bar{f} \quad (12)$$

where

$$\begin{aligned} a &= \begin{bmatrix} 0 & I \\ I & c \end{bmatrix}, & b &= \begin{bmatrix} -m^{-1} & 0 \\ 0 & k \end{bmatrix} \\ y &= \begin{bmatrix} p \\ x \end{bmatrix}, & f &= \begin{bmatrix} 0 \\ f_x \end{bmatrix} \end{aligned} \quad (13)$$

where p is the momentum coordinate vector and x is the displacement coordinate vector. Complex free-interface modes are determined by the homogeneous equation

$$a\dot{y} + by = 0 \quad (14)$$

where y is taken to have the form

$$y = \psi e^{\lambda t} \equiv \begin{bmatrix} \psi_p \\ \psi_x \end{bmatrix} e^{\lambda t} \quad (15)$$

Only the case of symmetric damping matrix, c , is considered. Then orthogonality and normalization of the modes leads to the equations

$$\psi^T a \psi = I, \quad \psi^T b \psi = -\Lambda \quad (16)$$

The x and p -partitions of rigid-body modes are defined by the equations

$$kx = 0, \quad km^{-1}p = 0 \quad (17)$$

Convergence is improved by supplementing a truncated set of complex modes with attachment modes, which are defined by the equation

$$\begin{bmatrix} -m^{-1} & 0 \\ 0 & k \end{bmatrix} \begin{Bmatrix} \phi_p \\ \phi_x \end{Bmatrix}_a = \begin{Bmatrix} 0 \\ f_x \end{Bmatrix} \quad (18)$$

After extensive manipulation of the attachment modes, the substructure response is approximated by a truncated set of complex modes, $\psi_{\bar{n}}$, and a set of residual attachment modes, ϕ_m , by the equation

$$y_a = \psi_{\bar{n}} z_{\bar{n}} + \phi_m z_m \quad (19)$$

where y_a is the approximated response.

When two or more substructures are combined to form a synthesized structure, compatibility equations must be enforced at substructure interfaces. These take the form of

$$E_1 X = 0 \quad (20)$$

where X is the union of substructure displacement vectors. Compatibility is also imposed on the velocity vector \dot{X} , and the combined compatibility equation is written in the form

$$\begin{bmatrix} -E_1 M^{-1} & 0 \\ 0 & E_1 \end{bmatrix} \begin{Bmatrix} P \\ X \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (21)$$

or

$$EY = 0 \quad (22)$$

Finally, the Y coordinates are expressed in terms of modal coordinates through

$$Y = T_1 Z \quad (23)$$

and the Z coordinates expressed in terms of independent system coordinates through

$$Z = SU \quad (24)$$

T_1 may contain either truncated sets of substructure complex modes or truncated sets of complex modes plus sets of attachment modes. Details of the derivation of the coupling matrix S for these cases are given in the dissertation. In the former case the final coupled system equation of motion is

$$(S^T S) \dot{U} - (S^T \Lambda S) U = S^T T_1^T F \quad (25)$$

The Hamiltonian approach outlined above employs the displacement vector and momentum vector as coordinates in the first-order equations of motion. The dissertation also briefly describes a state-vector approach to substructure coupling, but this is further developed in Ref. (14), which is abstracted below.

Finally, the dissertation contains a chapter on System Identification which describes a procedure for employing experimentally-identified frequencies, damping factors and mode shapes of lightly-damped systems to obtain substructure equations of motion transformed to modal coordinates, i.e. to form $\Psi^T a \Psi$ and $\Psi^T b \Psi$.

A number of simulation studies are presented in the dissertation. Table 5 presents a portion of the results of one of these studies. It shows the improvement in convergence resulting from the use of residual attachment modes.

Damped Frequency (ω_d)					
Mode No.	Truncation	Error	Normal Modes and Residual Attachment Modes	Error	Exact
	($N^\alpha=8, N^\beta=6, N^t=12$)	%	($N^\alpha=7, N^\beta=5, N^t=12$)	%	($N^t=18$)
1	0.230647	3.09	0.223737	0.00	0.223737
2	0.633534	2.70	0.616879	0.00	0.616877
3	1.227306	1.42	1.210142	0.00	1.210134
4	2.133363	6.46	2.003981	0.00	2.003861
5	3.010846	0.30	3.002025	0.00	3.002001
6	4.457241	5.76	4.215497	0.03	4.214322
7	5.879491	4.06	5.651209	0.03	5.649804
8	7.360751	2.19	7.304950	0.06	7.300932
9	10.17425	11.99	9.103936	0.21	9.084464
10	12.14582	0.18	12.19040	0.19	12.16735
11	15.80521	8.08	14.91669	2.01	14.62337
12	19.15414	7.76	21.25394	19.57	17.77519

Table 5. Comparison of Damped Frequencies for Clamped-Clamped Beam with Non-Proportional Damping - 12 D.O.F. System

In summary, the following conclusions were reached in this dissertation:

1. "A generalized first-order Hamiltonian formulation and a generalized state vector representation have been developed for carrying out component mode synthesis for viscously-damped systems. The component modes are complex for systems with non-proportional damping. Both the Hamiltonian approach and the state vector approach produce the same results. However, the state vector representation is more efficient since it avoids the inversion of the mass matrix. The state vector formulation is also easier to implement when experimental data is employed."
2. "Complex residual attachment modes, which are based on static approximation of the neglected higher modes, have been defined and derived. The inclusion of complex residual modes substantially improves the approximation to the system eigenvalues and eigenvectors in comparison with a solution based on truncated sets of component normal modes only."
3. "The state vector formulation of substructure coupling has been employed to identify the system characteristics of a complex structure using data from simulated substructure tests. A multiple-degree-of-freedom curve-fit algorithm is used to obtain the poles and residues of the substructures, and light damping assumptions are employed in estimating the complex mode shapes, generalized mass, damping and stiffness of the substructures. The system results obtained by substructure coupling show that excellent results can be obtained for lightly damped systems."

The following recommendations were offered:

"As a result of the work described in this dissertation, further research on the identification of the generalized mass, damping and stiffness for damped systems with complex modes is necessary in order to provide a more general and accurate system analysis technique via component mode synthesis. The effect of frequency resolution on the response functions needs to be investigated. A multiple-reference-multiple-degree-of-freedom curve fitting algorithm which curve-fits several frequency response functions simultaneously should be studied in order to improve the estimation of modal parameters. Finally, the relationship between the residual attachment modes described in Chapter 3 and frequency response function residuals obtained in conjunction with multiple-degrees-of-freedom curve fitting needs to be elucidated."

A Generalized Substructure Coupling Procedure for Damped Systems (Ref. 12)

This paper summarizes the portions of the dissertation, (Ref. 11), discussed above which deal with substructure coupling using truncated sets of complex modes. Beam and truss examples are given in the paper. The following conclusions are reached in the paper:

"A generalized substructure coupling procedure for systems with damping has been presented. Lagrange multipliers have been employed to incorporate the substructure interface compatibility constraints into the first-order system equations of motion. Complex free-interface substructure modes have been employed in two example problems. Although the coupling method is shown to produce the correct system modes if all substructure modes are retained, the results obtained for truncated systems are not very accurate. Since this is most likely a consequence of using

free-interface modes, further research is needed to explore convergence when other types of substructure modes are employed in the generalized substructure coupling procedure."

State Vector Formulation of Substructure Coupling for Damped Systems (Ref. 14)

This paper expands on the brief introduction to state vector formulation in Ref. 11. In the state vector formulation the substructure equation of motion still has the form of Eq. 12, but a , b , g and f are defined by

$$a = \begin{bmatrix} 0 & m \\ m & c \end{bmatrix}, \quad b = \begin{bmatrix} -m & 0 \\ 0 & k \end{bmatrix}, \quad y = \begin{bmatrix} v \\ x \end{bmatrix}, \quad f = \begin{bmatrix} 0 \\ f_x \end{bmatrix} \quad (26)$$

where v is the velocity vector. Free interface complex substructure modes are again determined by Eq. (14), where y is now taken to have the form

$$y = \psi e^{\lambda t} \equiv \begin{bmatrix} \psi_r \\ \psi_x \end{bmatrix} e^{\lambda t} \quad (27)$$

The matrix, Φ_m , of complex residual attachment modes is defined, and y is again approximated by y_a as in Eq. (19). The procedure for defining complex residual attachment modes requires the computation of all of the modal parameters of the substructure.

As in the case of the Hamiltonian formulation described above, compatibility equations are employed to formulate expressions for the coupling matrix, S , of Eq. (24). These expressions are presented for the case of coupling using complete or truncated sets of complex normal modes and for coupling using truncated normal modes plus residual attachment modes. Examples of these two approaches are provided in the paper. Table 6 illustrates the

results, which show the improved convergence of the method employing residual attachment modes.

Purely truncated modes		Truncated modes plus residual attachment modes	
$N_\alpha=8, N_\beta=6, N_t=12$		$N_\alpha=7, N_\beta=5, N_t=12$	
σ error %	ω_d error %	σ error %	ω_d error %
-5.31	3.09	0.00	0.00
5.62	2.70	0.00	0.00
-0.24	1.42	0.00	0.00
-0.67	6.46	0.00	0.00
-2.85	0.30	0.00	0.00
20.15	5.76	0.05	0.03
-22.87	4.06	-0.11	0.03
20.45	2.19	0.27	0.06
-0.58	11.99	-0.05	0.21
-5.63	0.18	1.23	0.19
54.66	8.08	-5.59	2.01
-31.70	7.76	18.84	19.57
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Table 6. Frequency and damping errors for a clamped-clamped beam with non-proportional damping

The conclusions stated in this paper are the following:

"A generalized substructure coupling procedure for systems with damping has been presented. A state vector formulation leading to a first-order system equation of motion has been developed. Complex residual attachment modes, which approximate the contribution of the neglected higher modes, are defined. A new method which employs incomplete complex normal modes in conjunction with the complex residual modes produces

significantly more accurate results than those obtained by using a truncated set of normal modes only. It is observed that the interface generalized coordinates can be eliminated from the final system coordinates by employing additional constraints on the generalized coordinates of attachment modes."

A Substructure Coupling Procedure Applicable to General Linear Time-Invariant Dynamic Systems (Ref. 15)

Although the substructure coupling studies of Refs. 11-14 demonstrated the use of first-order equations of motion, complex modes, and residual modes, they were subject to two significant limitations: the damping matrix was assumed to be symmetric, and all substructure modes were required in forming the truncated set of complex normal modes and the set of residual attachment modes. An alternative state vector formulation is derived in this paper, which has been submitted to the AIAA/ASME/ASCE/AHS 25th Structures, Structural Dynamics and Materials Conference.

The coupling procedure can be described as a generalized component mode synthesis technique. The equations of motion for each substructure are cast in state vector form to facilitate the formation of the substructure eigenproblem. If nonsymmetries exist in the mass, damping, or stiffness matrices, the eigenproblem will be non-self-adjoint. The low frequency right hand eigenvectors, along with a set of real attachment modes, are used as the Ritz expansion vectors. The concept of interface compatibility is used to couple the substructure equations of motion together to form the system equations of motion.

The synthesis technique is similar to the method presented in Reference 1, but with important differences. The attachment modes, as defined in Reference 1, are combinations of the high frequency modes not explicitly used as

Ritz vectors. This implies that the complete substructure eigenproblem must be solved - a potentially expensive task if the substructure contains a large number of degrees of freedom. The method to be presented avoids the problem of solving for the high frequency component modes by defining the attachment modes to be the static response of the substructure due to unit loads at the substructure interface. If the contribution of the low frequency component modes is removed from this static response, the so-called residual attachment modes are formed. The left-hand (adjoint) eigenvectors of the substructure are required to form the residual attachment modes.

The coupling procedure has been applied to structural systems containing combinations of rigid body modes, symmetric non-proportional damping, and non-symmetric damping matrices. Table 1 contains the eigenvalues obtained from a substructure synthesis compared to the exact values for the following structure,

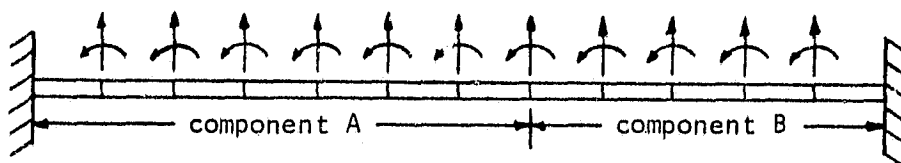


Figure 4. Structure Used in Skew-Symmetric Damping Study

The damping matrix for each substructure is skew-symmetric, and of the form

$$c \propto \begin{bmatrix} 5 & 1 & 1 & 1 \\ -1 & 5 & 1 & 1 \\ -1 & -1 & 5 & 1 \\ -1 & -1 & -1 & 5 \end{bmatrix}$$

Eigenvalue No.	Approx.		Exact		σ		ω_d		σ		ω_d		Mag	
	σ	ω_d	σ	ω_d	% Diff	ω_d	% Diff	σ	% Diff	ω_d	% Diff	Mag	%Diff	
1	-.607E-01	.245E-13	-.607E-01	0	-.001	0	0	-.001		0		-.001		
2	-.399E+00	-.159E-13	-.399E+00	0	-.003	0	0	-.003		0		-.003		
3	-.254E+00	.343E+00	-.254E+00	.343E+00	.025	.343E+00	.015	-.254E+00	.025	.015		.019		
4	-.254E+00	-.343E+00	-.254E+00	-.343E+00	.025	-.343E+00	.015	-.254E+00	.025	.015		.019		
5	-.337E+00	.770E+00	-.337E+00	.770E+00	.026	.770E+00	-.008	-.337E+00	.026	-.008		-.003		
6	-.337E+00	-.770E+00	-.337E+00	-.770E+00	.026	-.770E+00	-.008	-.337E+00	.026	-.008		-.003		
7	-.443E+00	.133E+01	-.441E+00	.133E+01	.356	.133E+01	.182	-.441E+00	.356	.182		.199		
8	-.443E+00	-.133E+01	-.441E+00	-.133E+01	.356	-.133E+01	.182	-.441E+00	.356	.182		.199		
9	-.573E+00	.203E+01	-.574E+00	.203E+01	-.222	.203E+01	.048	-.574E+00	-.222	.048		.028		
10	-.573E+00	-.203E+01	-.574E+00	-.203E+01	-.222	-.203E+01	.048	-.574E+00	-.222	.048		.028		
11	-.751E+00	.290E+01	-.738E+00	.288E+01	1.842	.288E+01	.455	-.738E+00	1.842	.455		.541		
12	-.751E+00	-.290E+01	-.738E+00	-.288E+01	1.842	-.288E+01	.455	-.738E+00	1.842	.455		.541		
13	-.910E+00	.394E+01	-.922E+00	.390E+01	-1.370	.390E+01	.982	-.922E+00	-1.370	.982		.859		
14	-.910E+00	-.394E+01	-.922E+00	-.390E+01	-1.370	-.390E+01	.982	-.922E+00	-1.370	.982		.859		
15	-.117E+01	.516E+01	-.114E+01	.516E+01	2.650	.516E+01	.073	-.114E+01	2.650	.073		.195		
16	-.117E+01	-.516E+01	-.114E+01	-.516E+01	2.650	-.516E+01	.073	-.114E+01	2.650	.073		.195		
17	-.109E+01	.684E+01	-.122E+01	.656E+01	-10.568	.656E+01	4.232	-.122E+01	-10.568	4.232		3.768		
18	-.109E+01	-.684E+01	-.122E+01	-.656E+01	-10.568	-.656E+01	4.232	-.122E+01	-10.568	4.232		3.768		
19	-.678E+01	.188E-06	-.113E+01	.832E+01	498.409	.832E+01	-100.000	-.113E+01	498.409	-100.000		-19.197		
20	-.102E+01	.881E+01	-.113E+01	-.832E+01	-9.868	-.832E+01	5.926	-.113E+01	-9.868	5.926		5.659		
21	-.102E+01	-.881E+01	-.659E+00	.940E+01	55.068	.940E+01	-6.272	-.659E+00	55.068	-6.272		-5.875		
22	.136E+02	.163E-06	-.659E+00	-.940E+01	*-.2E+04	-.940E+01	-100.000	-.659E+00	*-.2E+04	-100.000		43.877		
23	-.305E+02	.259E-06	-.705E+01	-.942E+01	331.832	-.942E+01	-100.000	-.705E+01	331.832	-100.000		161.988		

Eigenvalue = $\sigma + i\omega_d$

Table 7. Comparison of Approximate and Exact Eigenvalues for a 22 DOF (44 state Variables) Beam.
(Substructure A is represented by 15 Ritz vectors, and substructure B is represented by
12 Ritz vectors.)

As can be seen from Table 7, the method presented is very accurate even when nonsymmetries exist in the defining matrices, and the method of defining attachment modes is quite efficient from a computational point of view.

CONCLUSIONS AND RECOMMENDATIONS

Several significant contributions are contained in the reports and papers prepared under this contract and abstracted above. The principal conclusions of this work are:

1. The modal tuning procedures discussed in Refs. 6-8 provide a systematic procedure for combining modal data based on single-shaker FRF's to determine a system's undamped modes and frequencies.
2. The generalized multi-input, multi-output modal parameter estimation algorithm presented in Refs. 9 and 10 employs response data from several transducers, permits excitation at several points and of several types, and is capable of accurately identifying modal parameters from moderately "noisy" data.
3. Residual attachment modes significantly improve convergence in substructure coupling of damped systems. The generalized coupling procedure of Ref. 15 represents the culmination of the effort to develop a procedure capable of employing complex modes and having reasonable convergence properties.

The following recommendations are made for further work:

1. Development of the generalized multi-input, multi-output modal parameter estimation algorithm of Refs. 9 and 10 should be continued. An effort should be made to adapt the algorithm to a minicomputer-based modal analysis system, and the testing of the algorithm should be extended to modal testing of actual structures.
2. Development of the generalized substructure coupling procedure of Ref. 15 should be continued. A special effort should be made to determine appropriate experimental procedures for determining input data required by the method.

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